**Game Theory Final Study**

**Price of Anarchy**

PoA = worst case NE / Optimum

**Price of Stability**

PoA = best case NE / Optimum  
Upper bound 4/3

**Mixed Eq (M.1)**Includes probability distribution of all strategies

Ui() = SUM [Ui(A) \* i(A)] over all strategies  
 (A) = PROD [probability that player i would use strategy A] for all i

**Correlated Eq (M.1)**

For player 1, SUM [(A,B)\*(U1(A,B) - U1(A’,B))] >= 0  
For player 2, SUM [(A,B)\*(U2(A,B) - U2(A,B’))] >= 0  
Find a distribution where all of these are satisfied

**Coalition Majority Game (5.4)**

x1, x2, x3 >= 0

x1 + x2 >= 1

x1 + x3 >= 1

x2 + x3 >= 1

x1 + x2 + x3 = 1

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2(x1 + x2 + x3) >= 3 << sum equations w/ 2 vars  
2(x1 + x2 + x3) = 2 << equation with 3 vars

CONTRADICTION = NO CORE

**Stackelberg Equilibrium (M.2)**

Differentiate Pa and Pb profit functions, set = 0, solve to find Xa and Xb

If Xa is decided:

Differentiate Pb, set = 0, find equation for Xb

Differentiate Pa, plug in Xb equation, solve for Xa

Plug value into Xb

If Xa does not know Xb:

**Expected = P\*Xb1 + P\*Xb2**

Then do same as decided, but with expected

**When finding Xb, ensure to put both possible values**

**Local Connection Game (4.4)**

Prove that for alpha > n^2, all NE are trees

Start with NE for problem where alpha > n^2 that contains a cycle

Look at the cost vs benefit of adding the edge that created the cycle

Prove that cost > benefit

CONTRADICTION: Not a NE

**Global Connection Game (4.5)**

Cost to player i at NE <= SUM costEdgeWithChange  
 cost with any path <= #playersOnEdge \* SUM costEdgeAtSO  
 #playersOnEdge <= k  
 Therefore PoA bound is also k

**Atomic Congestion Game (4.2)**

Show NE need not exist:

Matrix of possible paths chosen by each player, show none are stable

**Φ: Atomic Admits Eq Flow (4.3)**

Given phi, assume player chooses a new strategy

Phi(f’) – phi(f) = SUM(e in p’/p) [term(f+ri) – term(f) + term(ri)]  
 + SUM(e in p/p’) [term(f-ri) – term(f) + term(ri)]

Since this is an affine function, we can simplify to: RESULT

**Φ: Load Balancing Admits Eq Flow (5.1)**

W2 = phi, S2 = phi, W1 = 1, S1 = 1

**Cost = max{w2/s1, w1/s2}** = w2/s1 = phi

**Opt = max{w2/s2, w1/s1}** = 1

Cost / Opt = phi

**Market Equilibrium (5.2)**

If each player has the same utility for all goods:

Price for all the goods is the same

Player gets a fraction of each good:

OPTIMUM:

* All goods are purchased
* Each player spends all its money

**Reverse Auction: Vickrey-Clarke-Groves (5.3)**

Winners = k bidders with minimum bid (Cost)

Price(i) = the lowest losing bid

**Pigou Bound (4.1)**

Show PoA is bounded by 4/3

Derive and set = 0 for sub-function in denominator  
 1 / (1-.25) <= 4/3